tion like exp $(-3\alpha z/2)$. Such a nonuniform relase of heat from the channel walls is automatically accomplished in practice with a steady-state flow provided the total cooling is sufficient to ensure wall temperatures close to absolute zero.

This study gives a sufficiently complete representation of the qualitative features of gas flow in a flat channel with cryogenic walls as confirmed by the results of numerical calculations for a typical range of the definitive parameters.

NOTATION

y, z, spatial coordinates; v_y, v_z, components of gas velocity; ρ, density; p, pressure; h, enthalpy; H, total enthalpy; ϰ, ratio of specific heats; τ_{yy}, τ_{yz}, τ_{zz}, components of viscous stress tensor; q_y, q_z, components of thermal flux vector; μ, coefficient of viscosity; σ, Prandtl number; R, channel half-width; q_1, q_2, q_3, flows of mass, momentum, and energy; α, dimensionless parameter characterizing axial flow variations; β, γ, dimensionless flow parameters [Eqs. (10)]; A_v, B_v, B_w, B_h, coefficients of expansions in the neighborhood of the wall; λ_f, coefficient of channel resistance; l, channel length; F_l, total resistive force of channel; S, cross-sectional area of channel; U, velocity at channel axis in entry cross section; 0, subscript denoting values on the axis.

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MATHEMATICAL SIMULATION OF HEAT- AND MASS-TRANSFER PROCESSES IN SEPARATED FLOWS WITH A LAMINAR MIXING REGION AT LOW REYNOLDS NUMBERS

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The lower limit of applicability of the mathematical model of heat and mass transfer constructed by Batchelor and Lavrent'ev for separated flow past a bottom trench is extended.

The mathematical model of heat and mass transfer for separated flow past a bottom trench constructed by Batchelor and Lavrent'ev [1] is of undisputed interest in many chemical-engineering problems. In particular, it is used successfully for study of hydrodynamic inhomogeneities in reactors with a fixed catalyst layer [2].

Quantitative estimates of heat and mass transfer are obtained in this model by employing dynamic and diffusion boundary-layer theory, so that its use is recommended only at Reynolds numbers exceeding hundreds or even thousands [3]. However, a significant number of processes take place at moderate or low Reynolds numbers, also with realization of a separation in the flow, so that this model could also be utilized. Numerical solutions of the Navier–Stokes equation by Myshenkov [4] for flow of a gas beyond a plate of finite thickness in the Reynolds number range of 1.7 to 100 show the possibility of existence of flows with separation at even small Reynolds numbers. The gas flow in the wake beyond the plate at Re < 1.7 is of a continuous nature, but at Re = 1.7 at the rear critical point there develops

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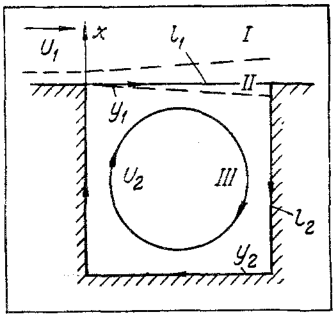

Fig. 1. Flow region diagram of [1] I, II, III) Regions of unperturbed flow, mixing, and turbulence formation.

a quite large region of low velocities, which with increase in Reynolds number transforms to a region of reverse current, producing separation of the flow.

Flow separation also sets in at relatively small Reynolds numbers beyond bodies of rotation. Kravchenko, Shevelev, and Shennikov have generalized results (including their own) of solutions of the Navier–Stokes equations for cases of flow past a cylinder of a viscous incompressible liquid under conditions of stationary and nonstationary (tending in the limit to stationary) motions or motion of the cylinder in the liquid [5]. They demonstrated that vortices first appear at values close to Re ~ 6. Taneda's [6] experimental data agree well with these observations. He measured the length of turbulent zones formed beyond a cylinder from streamline photographs for Reynolds numbers from 7 to 42. Transition from two- to three-dimensional flow (around a sphere) moves the critical turbulence formation zone from Re ~ 6 to Re ~ 24 [6]. The above demonstrates the desirability of expanding the lower limit of applicability of the separated-flow model for study of heat- and mass-transfer processes in chemical-engineering processes. A diagram of the model of [1] is shown in Fig. 1.

The basic equation for calculation of the forces acting on the outer limit of the boundary layer in the secondary flow was taken as

$$\int_0^{l_1} \tau(y_1)\,dy_1 = \int_0^{l_2} \tau(y_2)\,dy_2 \tag{1}$$

(this implies that the contribution of viscosity forces along a closed contour is equal to zero). For calculation of the forces acting in the mixing region [right side of Eq. (1)] of the primary and secondary flows, the solution obtained by Vulis and Kashkarov [7] for the case $Re \to \infty$, where the assumption of constant pressure over the entire flow field and, consequently, of fulfillment of the boundary-layer theory approximation of smallness of the transverse velocity component V in comparison to longitudinal component $U (V \ll U < U_\infty)$, produces no special objections. In flows with smaller Reynolds numbers it is possible that the condition grad P = 0 of [1] may be disrupted due to curvature of streamlines in flow past a body with discontinuity of the elements studied.

We must thus perform an approximate calculation of the contribution to interaction of the companion flows U_1 and U_2 (Fig. 1) of the pressure gradient ($P_1 \neq P_2$). For $x_1 \to -\infty$ and $x_2 \to +\infty$ the ratio of the static and velocity head gradients has the form [7]

$$\frac{P_1 - P_2}{\left[\frac{\rho (U_1 - U_2)^2}{2}\right]} = -4K. \tag{2}$$

Numerical evaluation of Eq. (2) for the condition $U_2/U_1 = 0$ ($U_2 = 0$, $U_1 > 0$) performed in [7] gives $4K \approx 0.0055$. With the assumption that K is little dependent on U_2/U_1 (for values of U_2/U_1 of practical interest), the result obtained indicates that at grad $P \neq 0$ the basic contribution to interaction is produced by tangent (viscous) force components applied to planes normal to x. This justifies a significant expansion of the range of applicability (over Reynolds number) of the function describing liquid motion in the laminar mixing region — Eq. (3) of [1].

In calculating the interaction forces between a liquid flow and a solid wall [left side of Eq. (1)], the small size of the Reynolds number proves to be more significant, and it is no longer possible to use the expression for total resistance coefficient calculated from boundary-layer theory as was done in [1]. In connection with this, the authors previously [8] introduced a correction for smallness of the Reynolds number by using an expression for resistance coefficient derived by Kuo Yung-hua, which has been well verified for Re $\geq$ 10 [3]. The relationship between boundary conditions, i.e., between values of the primary flow velocity U_1, velocities on the external boundary of the secondary flow U_2, and the geometric characteristics l_1 and l_2 (Fig. 1), then takes the form

$$\frac{l_2}{l_1} = 0.722m(m-1)^2 - \frac{10}{\mathrm{Re}_1}m - \frac{6.33}{\sqrt{\mathrm{Re}_1}}\sqrt{m}\sqrt{\frac{l_2}{l_1}}, \tag{3}$$

where $m = U_1/U_2$. One of the problems of further reducing the lower limit of applicability of the model considered is derivation of an expression for computation of the resistance coefficient for a flat lamina for flow-by of a viscous incompressible liquid at Reynolds numbers down to 1. To do this we solved the Oseen equation, results of which are presented below. The Oseen equation has the form [9]

$$\begin{gathered} U\frac{\partial U}{\partial x} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial x} + \nu\Delta U, \\ U\frac{\partial V}{\partial x} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial y} + \nu\Delta V, \\ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \end{gathered} \tag{4}$$

The fact that the resistance of a plane lamina flowed over by a viscous incompressible liquid is produced solely by friction forces allows us to limit our examination of equations for the flow function. Writing Eq. (4) in vector form, taking the curl of both sides, and introducing the flow function, we obtain

$$\left(U\frac{\partial}{\partial y} - \nu\Delta\right)\Delta\psi = 0 \tag{5}$$

(the continuity equation is satisfied automatically).

As is well known [10], the optimum coordinates for study of flow past a semiinfinite lamina are the coordinates of a parabolic cylinder (Fig. 2). The Laplace operator and differentiation operators in these coordinates have the form [11]

$$\begin{gathered} \Delta = \frac{1}{\sigma^2+\tau^2}\left(\frac{\partial^2}{\partial\sigma^2} + \frac{\partial^2}{\partial\tau^2}\right), \\ \frac{\partial}{\partial y} = \frac{1}{\sigma^2+\tau^2}\left(-\sigma\frac{\partial}{\partial\sigma} + \tau\frac{\partial}{\partial\tau}\right), \\ \frac{\partial}{\partial x} = \frac{1}{\sigma^2+\tau^2}\left(\tau\frac{\partial}{\partial\sigma} + \sigma\frac{\partial}{\partial\tau}\right). \end{gathered} \tag{6}$$

Substituting Eq. (6) in Eq. (5), we have

$$\left[U\left(-\sigma\frac{\partial}{\partial\sigma} + \tau\frac{\partial}{\partial\tau}\right) - \nu\frac{1}{\sigma^2+\tau^2}\left(\frac{\partial^2}{\partial\sigma^2} + \frac{\partial^2}{\partial\tau^2}\right)\right]\frac{1}{\sigma^2+\tau^2}\left(\frac{\partial^2}{\partial\sigma^2} + \frac{\partial^2}{\partial\tau^2}\right) = 0. \tag{7}$$

We write the boundary conditions for Eq. (7):

$$\begin{gathered} \sigma = 0 \quad \psi = 0 \quad \frac{\partial\psi}{\partial\sigma} = 0, \\ \sigma\to\infty \; \frac{\partial\psi}{\partial\sigma}\to\tau U \;\text{ or }\; \frac{\partial\psi}{\partial\tau}\to\sigma U. \end{gathered} \tag{8}$$

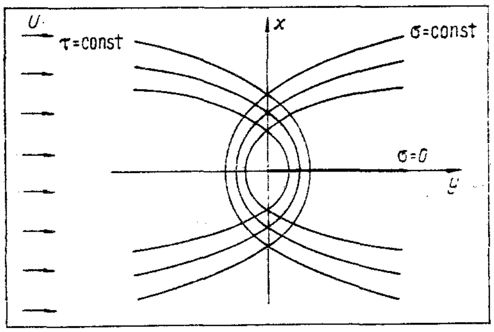

Fig. 2. Diagram of flow past a semi-infinite plate by a viscous incompressible liquid (in parabolic cylinder coordinates).

Considering the form of the boundary conditions (direct proportionality between the flow function and τ) and the fact that σ and τ appear in Eq. (7) only in even powers, we will seek a solution in the form of the following asymptotic series:

$$\psi \sim \tau f_0(\sigma) + \frac{1}{\tau} f_1(\sigma) + \ldots . \tag{9}$$

In the one-term approximation, substituting Eq. (9) in Eq. (7), we obtain for $f_0(\sigma)$

$$\frac{\nu}{U} f_0^{IV} + \sigma f_0^{III} + f_0^{II} = 0. \tag{10}$$

For Eq. (10), from Eq. (8) there follow the boundary conditions

$$\begin{gathered} \sigma = 0 \;\; f_0 = f_0^{I} = 0, \\ \sigma \to \infty \;\; f_0^{I} \to U. \end{gathered} \tag{11}$$

We denote $f_0^{II} = g$, $\nu/U = \lambda$. Then

$$\lambda g^{II} + \sigma g^{I} + g = 0. \tag{12}$$

We introduce the variable $\bar{\sigma} = (1/\sqrt{\lambda})\sigma$. Then

$$\begin{gathered} \frac{d}{d\sigma} = \frac{1}{\sqrt{\lambda}} \cdot \frac{d}{d\bar{\sigma}}, \quad \frac{d}{d\sigma^2} = \frac{1}{\lambda} \cdot \frac{d^2}{d\bar{\sigma}^2}, \\ \frac{d^2 g}{d\bar{\sigma}^2} + \bar{\sigma}\frac{dg}{d\bar{\sigma}} + g = 0. \end{gathered} \tag{13}$$

The solution of Eq. (13) has the form [12]

$$g = \exp(-\bar{\sigma}^2/4) D_0(\bar{\sigma}),$$

where $D_0(\bar{\sigma})$ is a parabolic cylinder function of index 0. Two linear independent solutions of Eq. (13) have the form [13]

$$g_1 = D_0(\bar{\sigma}) \exp(-\bar{\sigma}^2/4), \tag{14a}$$

$$g_2 = D_1(i\bar{\sigma}) \exp(-\bar{\sigma}^2/4). \tag{14b}$$

For g_1 we have [13]

$$g_1 = \exp(-\bar{\sigma}^2/4) D_0(\bar{\sigma}) = \exp(-\bar{\sigma}^2/4)\exp(-\bar{\sigma}^2/4)\; H_0\left(\frac{\bar{\sigma}}{\sqrt{2}}\right) = \exp\left(-\frac{1}{2}\cdot\frac{U}{\nu}\sigma^2\right), \tag{15a}$$

where $H_0(\bar{\sigma}/\sqrt{2}) = 1$ is a zero-order Hermite polynomial [13]. Then g_2 may be written in the form [14]

$$g_2 = \exp\left(-\frac{\bar{\sigma}^2}{2}\right) \int_0^{i\frac{\bar{\sigma}}{\sqrt{2}}} \exp(-t^2)\, dt. \tag{15b}$$

Thus four linearly independent solutions of Eq. (10) have the form

$$f_0^{(1)} = 1,\ f_0^{(2)} = \sigma, \tag{16a}$$

$$f_0^{(3)} = \int_0^{\bar{\sigma}} d\xi \int_0^{\xi} \left(-\frac{t^2}{2}\right) dt, \tag{16b}$$

$$f_0^{(4)} = \int_0^{\bar{\sigma}} d\eta \int_0^{\eta} \exp\left(-\frac{\xi^2}{2}\right) d\xi \int_0^{i\frac{\xi}{\sqrt{2}}} \exp(-t^2)\, dt. \tag{16c}$$

We write the general solution

$$f_0(\sigma) = C_1 + C_2\sigma + C_3 f_0^{(3)} + C_4 f_0^{(4)}, \tag{17}$$

where C_i are constants which will be determined below from the boundary conditions, and $f_0^{(3)}$ and $f_0^{(4)}$ are given by Eqs. (16b) and (16c). Requiring that the solution Eq. (17) satisfy the boundary conditions, we obtain the following system of equations:

$$C_1 + C_3 f_0^{(3)}(0) + C_4 f_0^{(4)}(0) = 0, \tag{18a}$$

$$C_2 + C_3 f_0^{(3)\prime}(0) + C_4 f_0^{(4)\prime} = 0, \tag{18b}$$

$$C_2 + C_3 f_0^{(3)\prime}(\infty) + C_4 f_0^{(4)\prime}(\infty) = U. \tag{18c}$$

Since $f_0^{(3)}(0) = f_0^{(4)}(0) = 0$, $f_0^{(3)\prime}(0) = f_0^{(4)\prime}(0) = 0$, it follows from Eqs. (18a) and (18b) that $C_1 = 0$ and $C_2 = 0$. Inasmuch as $f_0^{(4)\prime}(\infty) = \infty$, i.e., $f_0^{(4)\prime}$ diverges logarithmically at infinity, $C_4 = 0$. Then Eq. (18c) gives

$$C_3 f_0^{(3)\prime}(\infty) = U. \tag{19}$$

According to [12], we have

$$f_0^{(3)\prime}(\infty) = \frac{1}{\sqrt{\lambda}} \int_0^{\infty} \exp(-t^2/2)\, dt = \sqrt{\frac{\pi}{2\lambda}}.$$

Thus it follows from Eq. (19) that

$$C_3 = \sqrt{\frac{2\nu U}{\pi}}. \tag{20}$$

Substituting Eq. (20) in Eq. (17) and considering that $C_1 = C_2 = C_4 = 0$, we have

$$f_0(\sigma) = \sqrt{\frac{2}{\pi}} (\nu U)^{1/2} \int_0^{\sigma/\sqrt{\lambda}} d\xi \int_0^{\xi} \exp\left(-\frac{t^2}{2}\right) dt. \tag{21a}$$

Multiplying Eq. (21a) by τ, we obtain the flow function in the one-term approximation:

$$\psi = \sqrt{\frac{2}{\pi}} (\nu U)^{1/2} \tau \int_0^{\sigma\sqrt{U/\nu}} d\xi \int_0^{\xi} \exp\left(-\frac{t^2}{2}\right) dt. \tag{21b}$$

Thus a solution has been obtained for the Oseen equation for the flow function in the one-term approximation. Using this solution, we then find the lamina resistance coefficient

$$C_f = \frac{\mu \left.\frac{\partial U}{\partial x}\right|_{x=0}}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U^2}\cdot\frac{\partial}{\partial x}\left.\psi_x\right|_{x=0} = \frac{2\mu}{\rho U^2}\left.\psi_{xx}\right|_{x=0} \qquad (22)$$

In parabolic coordinates for $\partial^2/\partial x^2$ in accordance with Eq. (6) we have

$$\frac{\partial^2}{\partial x^2} = \frac{1}{\sigma^2+\tau^2}\left(\tau\frac{\partial}{\partial\sigma}+\sigma\frac{\partial}{\partial\tau}\right)\frac{1}{\sigma^2+\tau^2}\left(\tau\frac{\partial}{\partial\sigma}+\sigma\frac{\partial}{\partial\tau}\right) \approx \frac{1}{\tau^2}\cdot\frac{\partial^2}{\partial\sigma^2}$$

(considering that $\tau >> \sigma$, $\partial/\partial\sigma >> \partial/\partial\tau$). Thus,

$$\left.\psi_{xx}\right|_{x=0} \approx \frac{1}{\tau^2}\cdot\frac{\partial^2}{\partial\sigma^2}\left.\psi\right|_{\sigma=0} = \frac{1}{\tau^2}\cdot\left.\frac{\partial^2\psi}{\partial\sigma^2}\right|_0 .$$

We calculate $\psi_{\sigma\sigma}$ by differentiating the integral of Eq. (21a) at its upper limit:

$$\frac{\partial\psi}{\partial\sigma} \sim \sqrt{\frac{2}{\pi}}(\nu U)^{1/2}\tau\sqrt{\frac{U}{\nu}}\int_0^{\sigma\sqrt{\frac{U}{\nu}}}\exp\left(-\frac{t^2}{2}\right)dt = \sqrt{\frac{2}{\pi}}\tau U\int_0^{\sigma\sqrt{\frac{U}{\nu}}}\exp\left(-\frac{t^2}{2}\right)dt,$$

$$\frac{\partial^2\psi}{\partial\sigma^2} = \sqrt{\frac{2}{\pi}}\tau U\sqrt{\frac{U}{\nu}}\exp\left(-\frac{1}{2}\sigma^2\frac{U}{\nu}\right) = \sqrt{\frac{2U^3}{\pi\nu}}\exp\left(-\frac{\sigma^2 U}{2\nu}\right), \qquad (23)$$

$$\left.\psi_{\sigma\sigma}\right|_{\sigma=0} = \sqrt{\frac{2U^3}{\pi\nu}}\tau, \quad \left.\psi_{xx}\right|_{x=0} = \frac{1}{\tau}\left(\frac{2U^3}{\pi\nu}\right)^{1/2}.$$

Substituting Eq. (23) into Eq. (22), we obtain the local surface-friction coefficient:

$$c_f = \frac{2/\sqrt{\pi}}{\sqrt{\mathrm{Re}}} \sim \frac{1.13}{\sqrt{\mathrm{Re}}}. \qquad (24)$$

Here $\mathrm{Re} = Uy/\nu$ is the local Reynolds number, calculated at a distance from the forward edge.

Having integrated Eq. (24), we obtain the mean value of surface friction on one side of the plate:

$$c_f = \frac{\int_0^y c_f dy}{y} \sim \frac{2.26}{\sqrt{\mathrm{Re}}}. \qquad (25)$$

Known solutions for flow over a semiinfinite plane plate by a viscous incompressible liquid (Blasius, Kuo Yung-hua) were obtained, as indicated above, on the basis of Prandtl boundary-layer theory (a solution based on direct integration of the full Navier–Stokes equations was attempted, but not completed, by Kochin [3]). The approach used herein differs from those above in that the Oseen equation lies at the base of the calculations. It is thus of interest to compare the above result — Eq. (25) — with those obtained previously: by Blasius, $c_f = 1.328/\sqrt{\mathrm{Re}}$ and by Kuo Yung-hua, $c_f = (1.328/\sqrt{\mathrm{Re}}) + (4.18/\mathrm{Re})$. Results of this comparison are presented in Fig. 3. The inapplicability of the Blasius solution to the flow regimes studied is obvious. The agreement at Re ~ 20 between the results of the present study and the work of Kuo Yung-hua, in which the number of the approximation and the range of the "boundary layer" are increased, can be considered reasonable. Solution of the problem considered here of flow over a plane plate on the basis of the Oseen equation in the two-term or higher approximations (which is methodologically more complex) would evidently shift the point of coincidence of c_f values into the area of larger Reynolds numbers.

With consideration of Eq. (25), the relationship between boundary conditions, i.e., between primary flow velocity U_1, velocity at the outer boundary of the secondary flow U_2, and the geometric characteristics l_2 and l_1, take the form

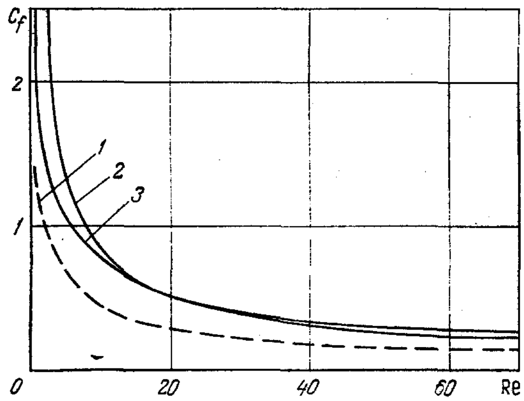

Fig. 3. Resistance coefficient of plane plate (far from edge) according to Blasius (1), Kuo Yung-hua (2), and the Oseen approximation of the present study (3).

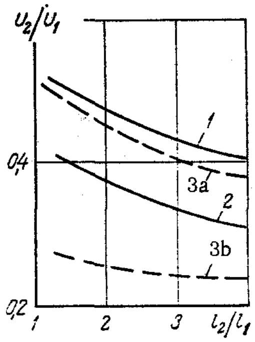

Fig. 4. Velocity ratio U_2/U_1 vs l_2/l_1 from Eq. (27) (1), Eq. (26) (2), Eq. (3) at Re = 1000 (3a), and Re = 10 (3b).

$$l_2/l_1 = 0.247m(m-1)^2. \tag{26}$$

The corresponding equation in [1] has the form

$$l_2/l_1 = 0.722m(m-1)^2. \tag{27}$$

The coefficient on the right side of Eq. (26) is almost three times smaller than that in Eq. (27). It is interesting to determine the contribution of this divergence to interaction of the primary and secondary flows. To do this we determine the limits of the geometric dimensions l_2/l_1, in which the flow pattern employed in [1] will act. Chang [15] indicates that developed turbulent flow (in contrast to flow with a diffusion of the mixing layer) in fine grooves is formed beginning approximately with h/b ~ 0.1, where h and b are the depth and width of the groove flowed over. In this case for a rectangular groove we have $l_2/l_1 \sim 1.2$. This quantity should obviously be taken as the lower limit. For the upper limit we may conditionally take an l_2/l_1 value close to 4 (which corresponds approximately to an h/b = 1.5-1 ratio), since it has been demonstrated several times [15-17] that with approach of h/b to 2 ($l_2/l_1 = 5$) generation of a second vortex in the groove takes place, so that the flow pattern and calculations used in the model are disrupted. For the indicated range of l_2/l_1 values the ratio U_2/U_1 was calculated with Eq. (26) and Eq. (27) (curves 2 and 1 of Fig. 4, respectively), and also by Eq. (3) for Re = 10 and 1000. The significant effect on the result of the form of the expression used in the model is evident. For flow regimes where Re $\leq$ 20, one should use Eq. (26), while for Re $\to \infty$, Eq. (27) is suitable. In the intermediate range, Eq. (3) should be employed. These recommendations should be followed in solving heat- and mass-transfer problems with the separated-flow model considered.

NOTATION

Re = Uy/ν, Reynolds number, where U is velocity; y, segment length; ν, coefficient of kinematic viscosity; τ_i, shear stress; P, pressure; l_1, l_2, lengths of mixing zone and cut wall; V, transverse component of liquid flow velocity; σ, τ, axes of parabolic cylinder coordinate system; ψ, flow function; C_f, resistance coefficient; h, b, depth and width of cut.

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INFLUENCE OF THE CONCENTRATION INITIAL SECTION ON THE MAGNITUDE OF DIFFUSION FLUXES IN TURBULENT FLUID FLOW

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UDC 532.517.4:536.2.242+629.7.063

Results of an experimental investigation of the influence of initial sections of a mass delivery surface on values of the Stanton number are presented.

A stabilized temperature (concentration) profile is achieved in very rare cases in turbulent fluid flows with Prandtl (Schmidt) numbers significantly greater than one (oil heat exchangers, electrochemical treatment in viscous electrolytes, etc.) and a length of the initial section of the transfer surface sufficiently large. Hence, the design dependences of the heat and mass transfer, obtained for a developed temperature (concentration) profile, are barely suitable under these conditions. The methods used at this time to take account of the influence of the initial thermal (diffusion) section on the average values of the Nusselt (Stanton) numbers [1, 2] are empirical in nature.

An experimental investigation was conducted in the range of Reynolds numbers between $1 \cdot 10^4$ and $1.3 \cdot 10^6$ and of Schmidt numbers, $1.5 \cdot 10^3 - 5.2 \cdot 10^4$, on two experimental setups. The change in the limit diffusion currents in an oxidation–reduction reaction on potassium ferroferricyanide which proceeds on solid electrodes is studied in the research as a function of the extent of the electrode along the flow. The method of conducting the tests and the experimental setups are described in detail in [3, 4]. The electrodes were glued flush with the side surface of a rotating cylinder. The rotating cylinder has definite advantages over other

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